Scaling Law for the Cosmological Constant from Quantum Cosmology with Seven Extra Dimensions

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Abstract According to a model of quantum cosmology the maximum number of degrees of freedom allowed in our three dimensions was determined by the size of seven extra dimensions in an initial excited state before inflation. The size of the extra dimensions can be inferred from a simple scheme for unifying the strong force and gravity. Coupled with the Bekenstein-Hawking entropy bound, these considerations lead to a scaling law for the cosmological constant that has been proposed independently by several authors.

Keywords Cosmological constant · Bekenstein-Hawking entropy · Quantum cosmology · Inflation

1 Introduction

In a simple quantum cosmology the Universe began with a quantum fluctuation from nothing into a homogeneous 10-dimensional space. That quantum state then tunneled into another state in which seven of the dimensions were excited, subsequently collapsing to their ground state. Certain parameters of the excited and ground states of the putative extra dimensions can be specified by considering the relative strengths of the gravitational and strong forces. In order to unify gravity and the nuclear force in the very early universe the gravitational force must have been much stronger at the beginning of inflation and it must have weakened during inflation. That change in the strength of the gravitational force may be modeled by allowing the Newtonian gravitational coupling G to vary. The size of the extra dimensions would correspond to the characteristic microscopic length scale associated with the gravitational coupling, and would decrease as gravity weakened [4].

In a simple unification scheme, the initial gravitational coupling G_i would have been such that

$$\frac{G_i m_n^2}{\hbar c} = 1, \tag{1}$$

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where m_n is the nucleon mass, $\hbar$ is the Planck constant and c is the speed of light in a vacuum. Equation (1) leads directly to

$$\frac{G_i}{G} = \frac{m_P^2}{m_n^2} \approx 1.7 \times 10^{38}, \tag{2}$$

where m_P is the Planck mass. As the seven compact dimensions collapsed from the excited state to the ground state, the gravitational coupling G_i must have decreased to G. In the excited state the size of the extra dimensions would have been roughly $l_i \equiv (\hbar G_i/c^3)^{1/2}$, which is the microscopic length associated with G_i. During inflation, as the gravitational coupling decreased to G, the size of the extra dimensions decreased to the Planck length l_P. Note that the size of the extra dimensions l_i in the excited state would have been, by (2), roughly equal to the Compton wavelength of the nucleon l_n [4].

In that scenario the available degrees of freedom associated with the seven extra dimensions were transferred to the three-dimensional universe during inflation. According to the Bekenstein-Hawking entropy bound, the maximum number of degrees of freedom allowed in a certain space is one fourth of the surface area of the space measured in Planck units [2]. The surface area $A_7(r)$ of a seven-dimensional sphere of radius r is given by

$$A_7(r) = \frac{16\pi^3}{15} r^6. \tag{3}$$

The actual area $A_7' \equiv f A_7$ of the seven-dimensional space that was part of the early Universe in its excited state may have been larger than the area in (3) by a positive, real factor f that represents possible convolutions of the surfaces. It is presumed here that f is of order near 10^0. If the characteristic radii of the extra dimensions were initially of order the nucleon's Compton wavelength l_n, the maximum number of degrees of freedom allowed in the initial seven-dimensional subspace would have been roughly

$$N_7 \sim \frac{A_7'(l_i)}{l_P^6} \sim f^6 \left(\frac{l_n}{l_P} \right)^6, \tag{4}$$

which is of the order $10^{119} f^6$ [5].

2 A Scaling Law for the Cosmological Constant

According to the model outlined by Mongan, N_7 should be also the maximum number of degrees of freedom that could be contained in our three dimensions [5]. Not only does that prediction agree with calculations of the maximum entropy of our Universe, but it also leads to a known scaling law for the cosmological constant. The maximum number of degrees of freedom that could ever be contained in our three dimensions is determined by the cosmological constant Λ. The de Sitter horizon R_Λ is the largest possible event horizon in a vacuum-dominated Universe, and it is equal to $c\sqrt{3/\Lambda}$. According to the Bekenstein-Hawking entropy bound, the maximum number N_Λ of degrees of freedom allowed in a sphere whose radius is the de Sitter horizon is

$$N_\Lambda = \frac{\pi R_\Lambda^2}{l_P^2} \sim \frac{c^5}{G\hbar\Lambda}. \tag{5}$$

If the current Hubble parameter is 71 km/s/Mpc and if the fraction of the cosmic energy inventory due to the vacuum is 0.73 then the cosmological constant is roughly $1.2 \times 10^{-35} \, \text{s}^{-2}$. With that value for Λ, the number N_Λ is roughly 2.5×10^{122}.

Given the uncertainty of a few orders of magnitude associated with the factor f and other unknown coefficients, the maximum number of degrees of freedom of our universe $N_\Lambda \sim 10^{122}$ agrees well with the number $N_7 \sim 10^{119}$ that follows from basic considerations of a simple quantum cosmological model. Furthermore, equating N_Λ to N_7 leads to a scaling relationship between the cosmological constant and other fundamental parameters of nature,

$$\Lambda \sim \frac{G^2 c^2 m_n^6}{\hbar^4}. \tag{6}$$

That is to say that the cosmological constant would be scaled to the sixth power of the nucleon mass, or vice versa. Remarkably, the scaling in (6) is identical to the scaling law proposed independently by Zel'dovich [6], Mena Marugan and Carneiro [3], and this author [1]. The fact that the model in [5] leads to a good prediction for the maximum entropy of our universe and a known scaling law for the cosmological constant is a compelling motivation to consider that both (6) and quantum cosmology with seven extra dimensions are physically meaningful.

References

1. Funkhouser, S.: Proc. R. Soc. A **462**, 2076 (2006)
2. Hawking, S.: Commun. Math. Phys. **43** (1975)
3. Mena Marugan, G., Carneiro, S.: Phys. Rev. D **65**, 087303 (2002)
4. Mongan, T.: Gen. Relativ. Gravit. **33**, 1415–1424 (2001)
5. Mongan, T.: Gen. Relativ. Gravit. **35**, 685–688 (2003)
6. Zel'dovich, Ya.B.: Pis'ma Zh. ETF **6**, 883 (1967)